HEAT TRANSFER DURING LAMINAR FLOW OF A VISCOELASTOPLASTIC MEDIUM IN A CIRCULAR TUBE

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An approximate analytical solution is obtained for the problem of heat transfer during the laminar flow of a viscoelastoplastic medium in a circular tube disregarding the energy dissipation.

The investigation of heat transfer during the flow of a viscoelastoplastic medium in a circular tube is of great practical interest, since such flow occurs in heat-transfer equipment and in transfer along the tubes of crosslinked dispersion systems.

The heat transfer during the flow of a viscoplastic medium described by the Shvedov–Bingham equation is investigated in [1]. Trusov and Tyabin [2] found that the laminar flow of an elastically compressible viscoplastic medium is described by the rheological equation

$$\tau_{rz}^2 = \theta^2 + \eta\tau_{rz}\frac{\partial V}{\partial r}.$$

The flow of a viscoelastoplastic medium in a circular tube breaks up into two zones: a zone of flow with constant velocity (elastic core of radius r_0) and a zone of gradient flow adjacent to the tube walls (Fig. 1). The distribution law of the flow velocity has the form

$$V(r) = \begin{cases} \dfrac{2\theta^2 l}{\eta\Delta p}\left[\dfrac{1}{2r_0^2}(R^2 - r_0^2) + \ln\dfrac{r_0}{R}\right], & 0 \leqslant r \leqslant r_0, \\ \dfrac{2\theta^2 l}{\eta\Delta p}\left[\dfrac{1}{2r_0^2}(R^2 - r^2) + \ln\dfrac{r}{R}\right], & r_0 < r \leqslant R, \end{cases}$$

where $r_0 = 2\theta l/\Delta p$. We shall investigate the heat-transfer problem under the following assumptions: the flow of the medium and the heat transfer become stabilized; the thermophysical characteristics of the medium are independent of the temperature; the temperature of the medium at the entrance to the heated section T_0 and the temperature of the tube wall T_w are constant and $T_0 \neq T_w$; there are no heat sources in the flow, the amount of heat appearing due to dissipation is negligibly small, and the heat flux along the z axis is insignificant compared to that along the radius of the tube. Under these assumptions, the energy equation is of the form

$$V(r)\frac{\partial T}{\partial z} = \frac{a}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \quad (1)$$

The boundary conditions are

$$T(0, r) = T_0, \ T(z, R) = T_w, \ \frac{\partial T(z, 0)}{\partial r} = 0. \quad (2)$$

The additional boundary conditions are determined by the coupling condition at the boundary of the elastic core:

$$T(z, r_0^-) = T(z, r_0^+), \ \frac{\partial T(z, r_0^-)}{\partial r} = \frac{\partial T(z, r_0^+)}{\partial r}.$$

By separation of variables, the boundary-value problem (1), (2) reduces to the Sturm–Liouville problem

$$\Phi''(y) + \frac{1}{y}\Phi'(y) + \varepsilon^2 f(y)\Phi(y) = 0, \quad (3)$$

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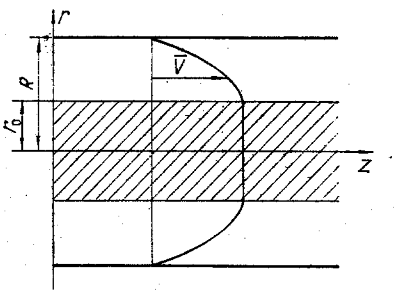

Fig. 1

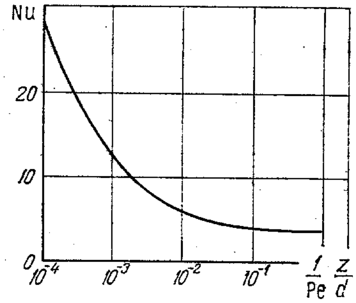

Fig. 2

Fig. 1. Change in the flow velocity of the medium over the cross section of the tube.

Fig. 2. Change in the local Nusselt number along the length of the tube.

$$\Phi'(0) = \Phi(1) = 0 \tag{4}$$

with the additional condition

$$\Phi(y_0^-) = \Phi(y_0^+), \quad \Phi'(y_0^-) = \Phi'(y_0^+).$$

In Eq. (3), f(y) denotes the dimensionless velocity of the medium and is given by the formula

$$f(y) = \begin{cases} 1, & 0 \leqslant y \leqslant y_0, \\ \dfrac{1}{V_0}\left[\dfrac{1}{2y_0^2}(1-y^2) + \ln y\right], & y_0 < y \leqslant 1, \end{cases}$$

where

$$V_0 = \frac{1}{2y_0^2}(1 - y_0^2) + \ln y_0.$$

We shall construct an asymptotic solution of problem (3), (4) by the method discussed in [1, 3, 4]. We thus obtain the following formulas:

$$c_n = \frac{\pi}{\alpha}\left(n + \frac{2}{3}\right), \tag{5}$$

$$\Phi_n(y) = \begin{cases} J_0(c_n y), & 0 \leqslant y \leqslant y_0, \\ \sqrt{\dfrac{2}{\pi c_n}}\,\dfrac{\cos\left[c_n \int\limits_0^y \sqrt{f(t)}\,dt\right]}{\sqrt[4]{y^2 f(y)}}, & y_0 < y < 0.5(1 + y_0), \\ (-1)^n \sqrt{\dfrac{2}{3}(1-y)}\, J_{\frac{1}{3}}\left[c_n \beta (1-y)^{\frac{2}{3}}\right], & 0.5(1 + y_0) \leqslant y \leqslant 1, \end{cases}$$

$$\alpha = \int\limits_0^1 \sqrt{f(t)}\,dt, \quad \beta = \frac{2}{3}\sqrt{\frac{1}{V_0}\left(\frac{1}{y_0} - 1\right)},$$

enabling us to obtain an infinite set of eigenvalues and eigenfunctions of problem (3), (4), which is of great significance for computing the heat transfer in viscoelastoplastic media for small reduced lengths of the tubes.

The temperature distribution of the medium in the tube has the form

TABLE 1. Eigenvalues and Constants in the Heat-Transfer Problem Computed for y_0 = 0.5

n	c_n	$(-1)^n d_n$	S_n	$(-1)^{n+1}\cdot\Phi_n'(1)$	n	c_n	$(-1)^n d_n$	S_n	$(-1)^{n+1}\cdot\Phi_n'(1)$
0	2,35927	1,56760	0,29431	1,04500	5	20,05376	0,37638	0,001996	2,13266
1	5,89816	0,85103	0,034695	1,41828	6	23,59265	0,33773	0,001366	2,25138
2	9,43706	0,62210	0,011588	1,65883	7	27,13155	0,30768	0,000986	2,35875
3	12,97596	0,50311	0,005512	1,84460	8	30,67045	0,28354	0,000741	2,45714
4	16,51486	0,42839	0,003140	1,99901	9	34,20935	0,26363	0,000574	2,54823
					10	37,74825	0,24688	0,000456	2,63323

$$\vartheta = \sum_{n=0}^{\infty} d_n \Phi_n \left(\frac{r}{R}\right) \exp\left(-\frac{2c_n^2}{\mathrm{Pe}}\cdot\frac{z}{d}\right),$$

where

$$\mathrm{Pe} = \frac{2\theta^2 l}{\eta \Delta p}\cdot\frac{V_0 R}{a}.$$

The d_n are determined from the boundary condition at the entrance to the heated segment. After some simple manipulations, we obtain

$$d_n = (-1)^n 2.28007 \beta^{\frac{1}{3}} \alpha^{-1} c_n^{-\frac{2}{3}}. \tag{6}$$

We write the change in the dimensionless mean mass temperature along the length of the tube in the form

$$\vartheta_{\mathrm{cp}} = \frac{1}{\int_0^1 y f(y)\, dy} \sum_{n=0}^{\infty} S_n \exp\left(-\frac{2c_n^2}{\mathrm{Pe}}\cdot\frac{z}{d}\right),$$

where

$$S_n = -d_n c_n^{-2} \Phi_n'(1), \tag{7}$$

$$\Phi_n'(1) = (-1)^{n+1} 0.72572 \beta^{\frac{1}{3}} c_n^{\frac{1}{3}}. \tag{8}$$

We compute the change in the local Nusselt number along the length of the tube from the formula (Fig. 2)

$$\mathrm{Nu} = -\frac{2}{\vartheta_m} \sum_{n=0}^{\infty} d_n \Phi_n'(1) \exp\left(-\frac{2c_n^2}{\mathrm{Pe}}\cdot\frac{z}{d}\right).$$

The limiting Nusselt number is

$$\mathrm{Nu}_\infty = 2c_0^2 \int_0^1 y f(y)\, dy.$$

The values of c_n, d_n, S_n, and $\Phi_n'(1)$, computed for y_0 = 0.5 from formulas (5)-(8) on an M-220 computer, are given in Table 1.

NOTATION

a, thermal diffusivity; c_n, eigenvalues; R, tube radius; d = 2R; T, temperature of the medium; T_0, input temperature; T_w, wall temperature; V(r), velocity; y = r/R, dimensionless radial coordinate; y_0, dimensionless radius of the elastic core; $\Delta p/l$, pressure drop along the length of the tube; η, dynamic viscosity coefficient; θ, limiting shear stress; τ, tangential stress; $\vartheta = (T - T_w)/(T_0 - T_w)$, dimensionless temperature; $\Phi_n(y)$, eigenfunctions; Pe, Peclet number; $J_0(y)$, $J_{1/3}(y)$, Bessel functions.

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HEAT TRANSFER IN GENERALIZED COUETTE FLOW OF A NONLINEAR VISCOPLASTIC FLUID

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The steady-state heat-transfer problem is solved for dissipative pressure flow of a nonlinear viscoplastic fluid between two parallel isothermal plates, one of which is moving at a constant velocity while the other is stationary.

Let us consider steady-state stabilized flow of a nonlinear viscoplastic fluid between two parallel infinite plates. The upper plate is moving in its own plane at a constant velocity U in the direction of the axis O_x. A constant pressure gradient grad p = A is present in the gap. The gradient can be of mechanical or other origin, such as a magnetic field moving along the channel axis and acting on a ferromagnetic suspension. The orientation of the velocity vector **U** can coincide with the direction of **A** or be opposite to it. This model of generalized Couette flow is valid, for example, for the description of fluid flow in the screw channels of an extruder. We consider the properties of the medium to be independent of the temperature. Constant temperatures are maintained on the plates: $T^{(1)}$ on the lower and $T^{(2)}$ on the ~~upper.~~

It has been shown [2] that three fully developed flow regimes are possible, depending on the rheological properties of the fluid, the magnitude and direction of the pressure gradient, and the velocity of the upper plate: 1) flow with a quasisolid zone (core) inside the main flow; 2) flow with the core adjacent to one of the plates; 3) flow without any core in the gap.

Accordingly, the equations of motion and thermal energy transport must be solved separately for the different zones and then matched at the interfaces (Fig. 1). Allowance must be made for the fact that dissipation of mechanical into heat energy takes place only in zones I and II, while in zone III the thermal conduction law for solids is realized.

To describe the rheological behavior of the fluid we use the generalized model [1]

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\mu_p \dot{\gamma})^{\frac{1}{m}} \tag{1}$$

with rheological parameters m, n, and μ_p (all real numbers).

Under the given initial assumptions, the problem is stated in the form

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{dy}, \tag{2}$$

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